



QUANTUM TRANSVERSE RELAXATION TIME

Sawsan Ahmed Elhoury Ahmed^{1*}, Mubarak Dirar Abdallah² & Asma Elbashir³

^{1*}University of Bahri - College of Applied & Industrial Sciences-Department of Physics Khartoum –Sudan

²International University of Africa- College of Science-Department of Physics & Sudan University of Science & Technology- College of Science-Department of Physics Khartoum-Sudan

³Sudan University of Science & Technology-College of Science-Department of Physics Khartoum-Sudan

Correspondence Author: sawsan.ahmed110@gmail.com

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Abstract

Transverse relaxation time T₂, plays an important role in MRI. It represents one of the important factors affecting image quality and the relaxation time. The aim of this work is to utilize the laws of quantum mechanics and electromagnetic theory to obtain new expressions for T₂.

Introduction

In nuclear magnetic resonance (NMR) spectroscopy and magnetic resonance imaging (MRI) the term relaxation describes how signals change with time. In general signals deteriorate with time, becoming weaker and broader. The deterioration reflects the fact that the NMR signal, which results from nuclear magnetization, arises from the over-population of an excited state. Relaxation is the conversion of this non-equilibrium population to a normal population. In other words, relaxation describes how quickly spins "forget" the direction in which they are oriented. The rates of this spin relaxation can be measured in both spectroscopy and imaging applications. The deterioration of an NMR signal is analyzed in terms of two separate processes, each with their own time constants. One process, associated with T₁, is responsible for the loss of signal intensity. The other process, associated with T₂, is responsible for the broadening of the signal. Stated more formally, T₁ is the time constant for the physical processes responsible for the relaxation of the components of the nuclear spin magnetization vector M parallel to the external magnetic field, B₀ (which is conventionally oriented along the z axis). T₂ relaxation affects the components of M perpendicular to B₀. In conventional NMR spectroscopy T₁ determines the recycle time, the rate at which an NMR spectrum can be acquired. Values of T₁ range from milliseconds to several seconds [1, 2, and 3].

Transverse relaxation time from time dependent perturbations theory

The time dependent perturbation theory provides scientists with powerful tool to find the probability that a certain energy level, for two energy levels system can be occupied at a certain time (t). This probability is obtained with the aid of the wave function, (C_m(t)), in the energy space the probability that the energy level in this occupation is given by [4, 5]:

$$|C_m(t)|^2 = \frac{4\pi^2}{\hbar^2} \mu_{mn}^2 (E_{0x})^2 t$$

Where μ_{mn} stands for the matrix element of the electric dipole moment P_{mn} which is given by:

$$P_{mn} = e\mu_{mn} = e \int \mu_m x \mu_n dx \quad (1)$$

It is straight forward to observe that at t = 0 the state m is completely empty, there for equation (1) reads:

$$|C_m(t = 0)|^2 = 0 \quad (2)$$

Where:

E = electric field intensity

\hbar = Planck's Constant

If immediately after t = 0 the state m is occupied till t = T₂, in this case equation (1) reads:

$$|C_m(t)|^2 = \frac{4\pi^2}{\hbar^2} \mu_{mn}^2 (E_{0x})^2 T_2 = 1 \quad (3)$$



Thus the relaxation time is given by:

$$T_2 = \frac{\hbar^2}{4\pi^2 E_{0x}^2 \mu_{mn}} \quad (4)$$

μ_{mn} Refers to electric polarization from state m to state n.

This expression is valid when the stimulating photons frequencies range from 0 - ∞ , if a photon of single frequency f which has energy approximate equal to the energy difference between the states m and n such that:

$$E_m - E_n \approx hf = \hbar\omega$$

$$\hbar\omega_m - \hbar\omega_n \approx \hbar\omega$$

$$\omega_m - \omega_n \approx 2\pi f$$

ω = angular frequency

$$\sin \frac{1}{2}(\omega_m - \omega_n - 2\pi f) \approx \frac{1}{2}(\omega_m - \omega_n - 2\pi f) \quad (5)$$

$$\frac{1}{2}(\omega_m - \omega_n - 2\pi f) \approx \text{a small angle}$$

Thus:

$$|C_m(t)|^2 = 4\mu_{mn}^2 E_{0x}^2 \text{Sin}^2 \frac{1/2(\omega_m - \omega_n - 2\pi f) t}{(\omega_m - \omega_n - 2\pi f)} = \frac{4\mu_{mn}^2 E_{0x}^2 (\omega_m - \omega_n - 2\pi f)^2}{4(\omega_m - \omega_n - 2\pi f)^2}$$

$$|C_m(t)|^2 = \mu_{mn}^2 E_{0x}^2 t \quad (6)$$

Again at $t = 0$

$$|C_m(t = 0)|^2 = 0 \quad (7)$$

i.e. the state m is empty. But at $t = T_2$ if it is fully occupied, i.e.

$$|C_m(t = T_2)|^2 = \mu_{mn}^2 E_{0x}^2 T_2 = 1 \quad (8)$$

The relaxation time can thus be given by:

$$\tau = T_2 = \frac{1}{\mu_{mn} E_{0x}^2} \quad (9)$$

But the square of electric field intensity is related to the photon density (n) and the photon energy $\hbar\omega$ via the relation

$$E_{0x}^2 = n\hbar\omega$$

$$T_2 = \frac{1}{\mu_{mn} n\hbar\omega} \quad (10)$$

If the energy of the photon excites protons between two states splitted by the interval due to the effect of the internal magnetic field B . Then:

$$\hbar\omega = g\mu_B M_S B_i \quad (11)$$

Hence:



$$T_2 \propto \frac{1}{B_i} \quad T_2 \propto \frac{1}{\omega} \quad (12)$$

g = constant (g factor)

μ_B = Bohr magnetron

m_s = Spin magnetic quantum number

B_i = internal magnetic field intensity

Transverse Relaxation Time and Conductivity in the Presence of Internal Local Field

When a photon which oscillates in electric and magnetic fields E and B are applied on a proton of mass (m), in the presence of a resistive force $F_r = \frac{mv}{\tau}$, $\tau = T_2$ and internal magnetic field B_i , the equation of motion thus becomes [6,7]:

$$m \frac{dv}{dt} = eE - \frac{mv}{T_2} + B_i ev + Bev \quad (13)$$

Where:

E = Electric field strength

e = Electron charge

v = velocity

m = mass

Assume the solution:

$$v = v_0 e^{i\omega t}, \quad E = E_0 e^{i\omega t}$$

By putting equation (14) into equation (13) one gets:

$$i\omega v_0 = eE_0 - \frac{mv_0}{T_2} + B_i e v_0 + B_e v_0$$

$$\left[\frac{m}{T_2} - B_i e - B_e + i\omega \right] v_0 = eE_0$$

$$v = v_0 e^{i\omega t} = \frac{e}{\frac{m}{T_2} - B_i e - B_e + i\omega} \underline{E}$$

The conductivity is thus given by:

$$J = ev = \frac{e^2}{\frac{m}{T_2} - B_i e - B_e + i\omega} E = \sigma E$$

Where:

J = Current density

σ = Electrical Conductivity

The conductivity can be spitted in to imaginary part and real part:

$$\sigma = \sigma_1 + i\sigma_2 = \frac{e^2 \left[\frac{m}{T_2} - B_i e - B_e - i\omega \right]}{\left[\frac{m}{T_2} - B_i e - B_e \right]^2 + [\omega]^2}$$



The conductivity is thus given by the real part:

$$\sigma_1 = e^2 \frac{\left[\frac{m}{T_2} - B_i e - B e - i m \omega \right]}{\left[\frac{m}{T_2} - B_i e - B e \right]^2 + [m \omega]^2} \quad (15)$$

When the conductivity is very low as in the case of the human body

$$\frac{m e^2}{T_2} = (B_i + B) e^3$$

Thus the relaxation time is given by:

$$T_2 = \frac{m}{(B_i + B) e} \quad (16)$$

$$T_2 \propto \frac{1}{B_i} \quad (17)$$

Conclusion

Reviewing equation (16) and equation (17) we found the value of the quantum transverse relaxation time.

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