

ANALITICAL STUDY OF DYAMIC CUTTING FORCE COMPONENTS IN FACE MILLING

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Abstract

For an understanding of the cutting dynamics of face milling, the knowledge of the cutting forces is one of the most fundamental requirements. This knowledge also gives very important information for cutter design, machine tool design, and detection of tool wear and breakage. This paper presents experimentally obtained cutting force components patterns for one revolution during face milling. Measurement of cutting force and results presentation are done by data acquisition system set by authors. The virtual instrument used for measuring the cutting force during face milling was set by use of graphical programming software such as matlab. Results of experiments are presented in graphical forms and mathematical model for cutting forces components are determined. Influence of cutting speed, feed, and depth of cut on the cutting force orthogonal components are analyzed.

Introduction

Manufacturing is an added value process that had always been of significant importance to human civilization. Machining operations comprise a substantialportion of the world's manufacturing infrastructure, making the enhancement andcontrol of metal removal processes one of the main concerns of the industry [1]. Due to the extensive use of highly automated machine tools in the industry, the manufacturing requires reliable models for the prediction of outputperformance of machining processes. The prediction of dynamic cutting forces playsan important role in the manufacturing industry. The focus of this paper is todevelop a reliable method to predict dynamic cutting force and their moments have significant importance in engineering technology and general inthe theory of material machining technology. Theyrepresent the basic categories of cutting mechanics.

Most of the researchers have dealt with development of force equation and the modeling of specific cutting pressure under the simplest conditions such as plane surface, limited consideration of cutter geometry, and no run out considerations. A force model which deals with more complicated machining situation should be systematically organized and computerized. In this view, general approach to develop a mechanistic face milling force model is presented by Kling and Devor[2]

Dynamic Force analysis and Simulation of Face milling operation

Initial vibration of the machine tool structure is caused by static cutting forces on the face milling tool leads to relative displacement between milling tool and work piece. This causes vibration in subsequent passes and results in variation in chip thickness and dynamic force component. Dynamic force component along with static force leads to regenerative vibration. Dynamic force models are formulated on following assumptions:

- 1. Static cutting forces begin to excite the stationary milling machine.
- 2. Dynamic cutting force is generated by regenerative vibration.

Dynamic Force Computation

The variation of the chip thickness based on double modulation principle [3,4] is defined by inner modulation corresponding to

relative displacement of system and by the outer modulation determined from inner modulation due to cutter rotational speed and

number of inserts [3,5]. For simplicity, to understand the dynamic system, assume two degree offreedom in X-direction for both

work piece and tool with instantaneous inner modulation of the **i th**insert at the cutter rotation angle **φ** is [2]

Where, \mathbf{X}_1 and \mathbf{X}_2 represents the displacements of the tool and work piece respectively.Instantaneous outer modulation \mathbf{X}_0 (i, φ) can be obtained by delaying the inner modulation by the time lag of one insert is [1]:

$$
X_o(i,\varphi) = X_1 \left[i \left(\left(\varphi - \left(\frac{2\pi N}{60 Z_n} \right) \right) \right) \right]
$$

Where, \mathbf{Z}_n is the total number of inserts. The instantaneous dynamic force component $\mathbf{dF}_x(\mathbf{i}, \mathbf{\omega})$ in the X - direction at a cutter rotation angle ϕ is defined as the product of cutting force coefficient, **K**_c, and incremental chip thickness variation as:

Where, \mathbf{Z}_n is the total number of inserts. The instantaneous dynamic force component $\mathbf{dF}_X(i, \varphi)$ in the X - direction at a cutter rotation angle **φ** is defined as the product of cutting force coefficient, **KC,** and incremental chip thickness variation as:

$$
dF_X(i,\varphi) = K_C U_X(i,\varphi)
$$

In the equation above equation, K**C** is complex frequency dependent quantity. However, for simplicity its average value is considered as static specific cutting pressure. Further, assuming a unit chip thickness, the resultant force with dynamic force **dFx**(i, ϕ) and static force **Fx** (i, ϕ) can be obtained by summation as:

$$
DF_X(i, \varphi) = F_X(i, \varphi) + dF_X(i, \varphi)
$$

By similarity, the formulation of dynamic force is extended three dimensional considerations. Let U_x (i, φ), U_y (i, φ), U_z (i, φ), U_z φ), which are incremental chip thicknesses in the X, Y, Z directions. Three dimensional resultant force **DFX** (i, φ), **DFY** (i, φ), **DF** z (i, φ) in three direction Cartesian coordinate system is obtained as follows:

$$
\begin{bmatrix} U_X \\ U_Y \\ U_Z \end{bmatrix} = \begin{bmatrix} G_{XX} & G_{XY} & G_{XZ} \\ G_{YX} & G_{YY} & G_{YZ} \\ G_{ZX} & G_{ZY} & G_{ZZ} \end{bmatrix} \begin{bmatrix} DF_X \\ DF_Y \\ DF_Z \end{bmatrix}
$$

Where, G is described as the ratio between incremental chip thickness variations to resultant cutting force. For the three dimensional modeling, assume tool and work piece as a rigid body. The system can be modeled as [2]:

$[K]$ \ddot{X} + $[C]$ \dot{X} + $[K]$ X = $[F]$

Where, $X = \begin{cases} x_1 & x_2 & y_1 & y_2 & z_1 & z_2 \end{cases}^T$

is the relative displacement vector between tool and work piece and are the force vectors.

$$
F = \begin{cases} F_x & 0 \quad F_y & 0 \quad F_z & 0 \end{cases}^T
$$

[M] is the mass matrix which depends on tool and the work piece, [C] is the damping coefficient depending on cutting feed and speed in between the tool and work piece and [K] is the stiffness connecting the tool and the work piece.

Using the above equation deflection in X (i), Y (i) and Z (i) for the ith insert can be computed using Wilson theta method [20]. To solve the equation for the first iteration, use static forces F_X , F_Y , F_Z as the force vector F, deflections X (i), Y (i) and Z (i) are computed. Knowing X (i), Y (i) and Z (i) which in turn compute U_X , U_Y , U_Z using equation (14). Dynamic forces DF_X, DF_Y, and DF_Z are then solved. In the next iteration, the computed dynamic forces are used to solve the equation (15). The procedure is continued until it converges.In Wilson Theta method, basic assumption is made such that the acceleration varies linearly not only with time for a period but also over an extended time period or step. Knowing the U_X, U_Y, U_Z , the resultant displacement of chip thickness variation can be obtained as:

$$
U_X(i,\varphi) = \left[X_O(i,\varphi) - X_I(i,\varphi) \right]
$$

U $(i, φ)$ can be expressed as:

$$
dF_r(i, \varphi) = K_R dF_t(i, \varphi)
$$

\n
$$
dF_t(i, \varphi) = K_c U(i, \varphi)
$$

\n
$$
dF_a(i, \varphi) = K_A dF_t(i, \varphi)
$$

$$
\begin{bmatrix} dF_x(\varphi) \\ dF_y(\varphi) \\ dF_z(\varphi) \end{bmatrix} = \sum_{i=1}^{Z_c} \begin{bmatrix} R_{iXX} & R_{iXY} & R_{iXZ} \\ R_{iYX} & R_{iYY} & R_{iYZ} \\ R_{iZX} & R_{iZY} & R_{iZZ} \end{bmatrix} \begin{bmatrix} U(i,\varphi) \end{bmatrix}
$$

where,

$$
R_{iXX} = \alpha_1 Sin\theta_i(\varphi) - \alpha_2 Cos\theta_i(\varphi)
$$

\n
$$
R_{iYY} = \alpha_1 Cos\theta_i(\varphi) + \alpha_2 Sin\theta_i(\varphi)
$$

\n
$$
R_{iYY} = K_R [\beta_1 Cos\theta_i(\varphi) + \beta_2 Sin\theta_i(\varphi)]
$$

\n
$$
R_{iYY} = K_R [\beta_1 Cos\theta_i(\varphi) + \beta_2 Sin\theta_i(\varphi)]
$$

$$
R_{iZX} = \alpha_3
$$

$$
R_{i\gamma Z} = K_A \left[\gamma_1 Cos \theta_i(\varphi) + \gamma_2 Sin \theta_i(\varphi) \right]
$$

 $R_{iXZ} = K_A \left[\gamma_1 Sin\theta_i(\varphi) - \gamma_2 Cos\theta_i(\varphi) \right]$

$$
R_{iZY} = K_R \beta_3 \quad R_{iZZ} = K_A \gamma_3
$$

Hence the instantaneous dynamic force at cutter rotation angle φ, **DFX** (i, φ), **DFY** (i, φ), **DFZ** (i, φ) can be obtained by adding the static force components **FX, FY** and **F^Z** and dynamic force components **dFX, dFY, dF^Z** as follows:

$$
\begin{bmatrix} DF_X \\ DF_Y \\ DF_Z \end{bmatrix} = \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} + \begin{bmatrix} dF_X \\ dF_Y \\ dF_Z \end{bmatrix}
$$

Verification of experiment: *The first part of the verification:* was conducted using experimental data taken from "Theoretical modelling and simulation of milling forces; X. P. Li, A.Y.C. Nee, Y.S. Wong, H.Q. Zheng." where given set of experimental values is correlated with the simulated values and its observed that it's a good agreement with the simulated results.

Table 1.1 below gives a set of machining condition. Table 1.2 shows the correlation between simulated experimental cutting forces

Table 1.1 Cutting condition

Fig 1.1 Simulated Static and Dynamic forces in time domain for 2 insert

It has been observed that the simulated values closely scrutinized with the measured force given in references. *The second part of the verification:* was conducted using experimental data taken from Fu, H., Devor, R.E., Kapoor, S.G., "A mechanistic model for the prediction of the force system in face milling operations". Table below shows the cutting condition.

Cutting conditions:

The table 1.4 below gives the comparison between simulated and experimental value.

Fig 6.2 Simulated forces along F^z for 7 inserts

Conclusions

This paper involves a procedure for the simulation of static and dynamic cutting forces in face milling operation. The mechanistic model which is selected for simulation and verification are the ones which takes in to account the initial position errors of the inserts and spindle eccentricity for the analysis. The relevant equations developed have been developed for simulating both static and dynamic forces. Dynamic cutting force equations are derived. Program has written using Wilson Theta method in a mat lab. The simulated forces, both static and dynamic are closely scrutinized with the measured force given in references.

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Program details
gammaL = 20 * 3.142/180;
gammaR = 14 * 3.142/180;
gammaA = 8 * 3.142/180;
alpha1 = 1;
alpha2 = - tan(gammaR);
alpha3 = -\tan(gammaA)/\cos(gammaR);
beta1 = cos(gammaL) * tan(gammaR)/cos(gammaA);beta2 = \cos(gamma_L)/\cos(gamma_A);beta3 = sin(gammaL)/(cos(gammaA) * cos(gammaR));
```

```
gamma1 = tan(gammaA)*cos(gammaL)/cos(gammaR);gamma2 = -\sin(gammaL)/\cos(gammaA);gamma3 = cos(gammaL)/cos(gammaR);
```
% All Input conditions are defined in the above set

%***8 % Initialization of static and dynamic forces $dynSumFX = 0$; $dynSumFY = 0$; $dynSumFZ = 0$; $sumFX = 0$; $sumFY = 0$; $sumFZ = 0$; %*** % All other input parameters are defined $n = 4$: $ft = 0.22$; theta $(4) = 0$; $d = 0.1$; theta $T = 0.000$; $Kt = 2500$; $Kr = 0.67$; $Ka = 0.375$; $rpm = 365;$ $E = 0.3$; $R = 50.8$; entryAngle = $22.5 * 3.142/180.0$; exitAngle = $180 * 3.142/180.0$; %*** % Axial and Radial Runouts $err(1) = 0.045$; $era(1) = 0.032$; $err(2) = 0.071;$ $era(2) = 0.080;$ $err(3) = 0.031$; $era(3) = 0.080$; $err(4) = 0.000;$ $era(4) = 0.020;$ $err(5) = 0.005;$ $era(5) = 0.016;$ $err(6) = 0.063;$ $era(6) = 0.024;$ $err(7) = 0.071;$ $era(7) = 0.024;$ $err(8) = 0.036$; $era(8) = 0.040$; %** handle = waitbar(0 , Performing Calculation...'); % Calculating the forces for about 1000 iterations forhai $= 1:1000$ $sumFX = 0$; $sumFY = 0$; $sumFZ = 0$; $dynSumFX = 0$; $dynSumFY = 0$; $dynSumFZ = 0$; phi = 1.095*hai; % getting the angle of cutter at every instant in radians per sec. for $i = 1:4$ angle $= 360/n$; theta(i) = (i-1) $*$ angle + phi; theta(i) = theta(i) $*$ 3.142/180.0; if theta(i) $>= 6.284$ theta(i) = mod(theta(i), 6.284); end

 $C1 = E * sin(theta(i));$ $D1 = R * sin(thetaT) * sin(theta(i));$ if $i > 1$ $C2 = (err(i) - err(i-1)) * cos(htetaT) * sin(hteta(i));$ $D3 = (era(i) - era(i-1)) * sin(thetaT);$ else $C2 = (err(i) - err(8)) * cos(htetaT) * sin(hteta(i));$ D3 = (era(i)- era(8)) $* sin(theta)$; end $C3 = \text{era}(i) * \text{sin}(\text{theta})$; $D1 = R * sin(thetaT) * sin(theta(i));$ $D2 = err(i) * sin(theta)$; $C = ft * sin(theta(i)) + C1 + C2 + C3;$ $D = d + D1 + D2 + D3;$ $A(i) = C * D;$ $Ft(i) = (2500 * A(i)) / (cos(gamma R) * cos(gamma A));$

 $Fr(i) = 0.67 * F(t)$; $Fa(i) = 0.375 * Ft(i);$

 $FT(i) = F(t) * cos(gamma A) * cos(gamma R) + Fr(i) * cos(gamma L) * sin(gamma R) + Fa(i) * cos(gamma L) *$ sin(gammaA);

 $FR(i) = -F(t_i) * cos(gamma A) * sin(gamma R) + Fr(i) * cos(gamma L) * cos(gamma R) - Fa(i) * sin(gamma L) *$ cos(gammaR);

 $FA(i) = -F(t) * sin(gamma A) + Fr(i) * sin(gamma L) + Fa(i) * cos(gamma A);$

```
if theta(i) >entryAngle& theta(i) <exitAngle
    delta = 1:
else
    delta = 0;
end
```

```
Fx = delta * (sin(theta(i)) * FT(i) - cos(theta(i)) * FR(i));Fy = delta * (cos(theta(i)) * FT(i) + sin(theta(i)) * FR(i));Fz = delta * FA(i);
```
 $sumFX = sumFX + Fx;$ $sumFY = sumFY + Fy$; $sumFZ = sumFZ + Fz$;

dynamic;

xInnerModulation(i) = $x(1,11) - x(2,11)$;

yInnerModulation(i) = $x(3,11) - x(4,11)$; zInnerModulation(i) = $x(5,11) - x(6,11)$; $xOuterModulation(i) = xInnerModulation(i)*(i*(phi - 2*3.14152*rpm/(60*3)));$ yOuterModulation(i) = yInnerModulation(i)*(i*(phi - 2*3.14152*rpm/(60*3))); zOuterModulation(i) = zInnerModulation(i)*(i*(phi - 2*3.14152*rpm/(60*3))); $uX(i) = xOuterModulation(i) - xInnerModulation(i);$ $uY(i) = yOuterModulation(i) - yInnerModulation(i);$ $uZ(i) = zOuterModulation(i) - zInnerModulation(i);$ $u(i) = sqrt((uX(i)^{2}) + (uY(i)^{2}) + (uZ(i)^{2}));$ $dFt(i) = Kt * u(i);$ $dFr(i) = Kr^*dFt(i);$ $dFa(i) = Ka* dFt(i);$ $Rxx = alpha1*sin(heta(i)) - alpha2*cos(heta(i));$ $Rxy = Kr[*](beta1[*]sin(theta(i)) - beta2[*]cos(theta(i)))$; $Rxz = Ka*(gamma1)*sin(theta(i)) - gamma2*cos(theta(i))$; $Ryx = alpha1*cos(theta(i)) + alpha2*sin(theta(i));$ $Ryy = Kr[*](beta1[*]cos(theta(i)) + beta2[*]sin(theta(i)))$; $Ryz = Ka*(gamma*cos(theta(i)) + gamma2*sin(theta(i)))$; $Rzx = alpha3;$ $Rzy = Kr^*beta3;$ $Rzz = Ka*gamma3;$ $dynSumFX = dynSumFX + delta * (Kt*(Rxx + Rxy + Rxz)*u(i));$ $dynSumFY = dynSumFY + delta * (Kt*(Ryx + Ryy + Ryz)*u(i));$ $dynSumFZ = dynSumFZ + delta * (Kt*(Rzx + Rzy + Rzz)*u(i));$ end $ang(hai) = phi;$ $dynFX(hai) = dynSumFX;$ $dynFY(hai) = dynSumFY;$ $dynFZ(hai) = dynSumFZ;$ $FX(hai) = sumFX;$ $FY(hai) = sumFY;$ $FZ(hai) = sumFZ;$ $totalForceX(hai) = FX(hai) + dynFX(hai);$ $totalForceY(hai) = FY(hai) + dynFY(hai);$ $totalForceZ(hai) = FZ(hai) + dynFZ(hai);$ waitbar((hai)/(1000),handle) end time = ang $*$ 0.5/1095; plot(time, FX) hold on;

plot(time, FY) hold on;

plot(time, FZ);

plot(time,totalForceX); hold on;

plot(time,totalForceY); hold on;

plot(time,totalForceZ); hold on;